Erratum of the paper

Topological properties of linearly coupled expanding maps lattices

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1 Proof of Proposition 3.2

The proof of Proposition 3.2 is not correct because there are sets in $\ell^\infty(\mathbb{Z})$ for which every one-dimensional canonical projection contains an interval, say I_s, but which do not cover the product of intervals $\bigotimes_{s\in\mathbb{Z}} I_s$. The image of $[0,1]^{\mathbb{Z}}$ by a convolution satisfying (H2) and (H3) is an example of such a set. This section is a proof, which we believe to be correct.

Proof: For each $i \in \{1, \cdots, N\}$, let $u^{(i)}$ be the middle point of $f(\mathrm{I}_i)$ and $\delta_i = \frac{|f(\mathrm{I}_i)|}{2}$, i.e.

$$f(\mathrm{I}_i) = u^{(i)} + [-\delta_i, \delta_i].$$

For each i, the condition $\mathrm{I}_j \subset \mathrm{Int} f(\mathrm{I}_i)$ implies the existence of $0 < \alpha_{i,j} < 1$ such that

$$I_j \subset u^{(i)} + [-\alpha_{i,j}\delta_i, \alpha_{i,j}\delta_i].$$

In other words, there exists $0 < \alpha < 1$ such that for any $\omega, \omega' \in \{1, \cdots, N\}^{\mathbb{Z}}$ so that $\mathrm{I}_{\omega'_s} \subset \mathrm{Int} f(\mathrm{I}_{\omega_s})$, $s \in \mathbb{Z}$, we have

$$\mathrm{I}_{\omega'} \subset u^\omega + \bigotimes_{s\in\mathbb{Z}} [-\alpha\delta_{\omega_s}, \alpha\delta_{\omega_s}] \subset F(\mathrm{I}_\omega),$$

where $u^\omega_s = u^{(\omega_s)}$ for every s. The left inclusion shows that one only has to show that if $\|\mathrm{Id} - L\|$ is sufficently small, we have

$$u^\omega + \bigotimes_{s\in\mathbb{Z}} [-\alpha\delta_{\omega_s}, \alpha\delta_{\omega_s}] \subset L \circ F(\mathrm{I}_\omega).$$

The latter is a consequence of the following result.

Lemma 1.1 *For any $\gamma > 1$, there exists $\varepsilon_\gamma > 0$ such that for any coupling satisfying $\|Id - L\| < \varepsilon_\gamma$, L^{-1} exists and for arbitrary $\omega = \{\omega_s\}_{s\in\mathbb{Z}} \in \{1, \cdots, N\}^{\mathbb{Z}}$, we have*

$$L^{-1}(\bigotimes_{s\in\mathbb{Z}} [\delta_{\omega_s}, \delta_{\omega_s}]) \subset \bigotimes_{s\in\mathbb{Z}} [-\gamma\delta_{\omega_s}, \gamma\delta_{\omega_s}].$$

Indeed, if $\|\mathrm{Id} - L\| < \varepsilon_\gamma$, then linearity implies that

$$L^{-1}(\bigotimes_{s\in\mathbb{Z}}[\alpha\delta_{\omega_s}, \alpha\delta_{\omega_s}]) \subset \bigotimes_{s\in\mathbb{Z}}[-\alpha\gamma\delta_{\omega_s}, \alpha\gamma\delta_{\omega_s}].$$

Let $1 < \gamma < \frac{1}{\alpha}$ and choose ε_γ smaller if necessary so that, in addition to the previous relation, the condition $\|\mathrm{Id} - L\| < \varepsilon_\gamma$ implies

$$|(L^{-1}u^\omega)_s - u_s^\omega| < (1-\alpha\gamma)\delta_{\omega_s}, \quad s \in \mathbb{Z}.$$

Consequently, for any L such that $\|\mathrm{Id} - L\| < \varepsilon_\gamma$, we have

$$L^{-1}(u^\omega + \bigotimes_{s\in\mathbb{Z}}[-\alpha\delta_{\omega_s}, \alpha\delta_{\omega_s}]) \subset u^\omega + \bigotimes_{s\in\mathbb{Z}}[\delta_{\omega_s}, \delta_{\omega_s}] = F(\mathrm{I}_\omega),$$

from which the desired result follows by applying L.

Proof of the Lemma: In all the proof, we assume that $\varepsilon_\gamma \leqslant 1$. As a consequence, L^{-1} exists and is a convolution. Let $\{\ell_n^{(-1)}\}_{n\in\mathbb{Z}}$ be the sequence representing L^{-1}. Assume that $\ell_0^{(-1)} > 0$ and let $\delta = \max_{1\leqslant i\leqslant N} \delta_i$. Take any $\omega \in \{1, \cdots, N\}^{\mathbb{Z}}$ and any $u \in \bigotimes_{s\in\mathbb{Z}}[\delta_{\omega_s}, \delta_{\omega_s}]$. We have

$$(L^{-1}u)_s \leqslant \ell_0^{(-1)}\delta_{\omega_s} + \delta \sum_{n\neq 0} |\ell_n^{(-1)}|, \quad s \in \mathbb{Z}.$$

We use this bound to show that

$$(L^{-1}u)_s \leqslant \gamma\delta_{\omega_s}, \quad s \in \mathbb{Z}, \tag{1}$$

where $\gamma > 1$ is given and provided that $\|\mathrm{Id} - L\|$ is sufficiently small.

Given $\gamma > 1$ and $i \in \{1, \cdots, N\}$, there exists $\eta_i > 0$ such that

$$(1+\eta_i)\delta_i + \delta\eta_i \leqslant \gamma\delta_i.$$

Now, we have

$$\ell_0^{(-1)} \leqslant \sum_{n\in\mathbb{Z}} |\ell_n^{(-1)}| = \|L^{-1}\| \leqslant \sum_{k\in\mathbb{Z}^+} \|\mathrm{Id} - L\|^k \leqslant \frac{1}{1-\varepsilon},$$

whenever $\|\mathrm{Id} - L\| \leqslant \varepsilon$. Consequently, there exists $\varepsilon_2 > 0$ such that $\|\mathrm{Id} - L\| \leqslant \varepsilon_2$ implies that

$$\ell_0^{(-1)} \leqslant \min_{1\leqslant i\leqslant N} 1 + \eta_i$$

Moreover, estimating the sequence $\{\ell_n^{(-1)}\}$ using the Neumann series defining L^{-1}, we obtain

$$\ell_0^{(-1)} \geqslant 1 - \sum_{k\in\mathbb{N}} \|\mathrm{Id} - L\|^k \geqslant \frac{1-2\varepsilon}{1-\varepsilon},$$

whenever $\|\mathrm{Id} - L\| \leqslant \varepsilon$. (This shows that $\ell_0^{(-1)} > 0$ when $\varepsilon < \frac{1}{2}$). Consequently, there exists $\varepsilon_3 > 0$ such that $\|\mathrm{Id} - L\| \leqslant \varepsilon_3$ implies that

$$\sum_{n\neq 0} |\ell_n^{(-1)}| = \|L^{-1}\| - \ell_0^{(-1)} \leqslant \frac{2\varepsilon_3}{1-\varepsilon_3} \leqslant \min_{1\leqslant i\leqslant N} \eta_i.$$

We conclude that the inequality (1) holds when $\|\mathrm{Id} - L\| < \varepsilon_\alpha = \min\{1, \frac{1}{2}, \varepsilon_2, \varepsilon_3\}$.

Using linearity, we obtain corresponding lower bound for $(L^{-1}u)_s$ and the Lemma follows. $\square$

2 Proof of Proposition 3.3

In the proof of Proposition 3.3, the intersections

$$\mathcal{F}_{f,L}^{-t}(\mathrm{I}_{\omega^{t+1}}) \cap \mathcal{F}_{f,L}^{-t+1}(\mathrm{I}_{\omega^{t}}) \neq \emptyset.$$

do not suffice to ensure that $\mathrm{J}_{\omega^1,\cdots,\omega^t} = \bigcap_{k=1}^{t} \mathcal{F}_{f,L}^{-k+1}(\mathrm{I}_{\omega^k})$ is non-empty. However, a classical argument shows that when the following inclusions hold (which is the case by Proposition 3.2)

$$\mathrm{I}_{\omega^{t+1}} \subset \mathcal{F}_{f,L}(\mathrm{I}_{\omega^t}), \quad t \in \mathbb{N},$$

every admissible cylinder $\mathrm{J}_{\omega^1,\cdots,\omega^t}$ is non-empty.